Symposium: Embodied Learning in Early Mathematics

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In this symposium we present some of the findings from Phase 1 of a three-phase project (2021-2024) titled *Embodied Learning in Early Mathematics and Science* (ELEMS). The project aims to translate embodied cognition research from the fields of neuroscience, psychology and education into evidence-based classroom teaching strategies, and to produce professional learning materials for teachers. The overall research design for the project is a three-phase structure, guided by design-based research principles and utilising mixed methods of data collection and analysis (Refer to Way & Ginns, 2022 for a project rationale). The underlying premise for the project is that the haptic modes (gesture, touch-tracing, body-movement and drawing) of embodied learning are under-utilised for mathematical representation, and as thinking and communicating tools in the development of mathematical understanding.

Phase 1 of the project involved a year-long collaboration with seven teachers in one NSW school, and their classes of Preschool to Year 2 children. The school has 340 students, with an additional 38 students in an attached preschool. The students come from a diverse range of cultures and 78% of students are from Non-English-Speaking Backgrounds (NESB). The researchers supported the teachers in their explorations of interpreting the research-based key ideas about embodied learning provided by the researchers, into teaching-learning activities for their students. Each of the three papers in this symposium reports a specific aspect selected from the broad range of research outcomes.

Paper 1: Connecting Mathematical Processes and Conceptual Body Movement—Katherin Cartwright & Jennifer Way

Paper 2: Finger Tracing, Noticing Structures and Drawing—Jennifer Way & Katherin Cartwright

Paper 3: Changes in Year 2 Children's Drawings of a Subtraction Story—Jennifer Way & Katherin Cartwright

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The University of Sydney research team: Dr Jennifer Way (Mathematics), Dr Paul Ginns (Educational Psychology), Dr Christine Preston (Science), Dr Amanda Niland (Early Childhood), Dr Jonnell Upton (TESOL), Dr Katherin Cartwright (Mathematics).

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Connecting Mathematical Processes and Conceptual Body Movement

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Conceptual body movement in education is an external representation mode that research suggests can support children's learning about mathematical phenomena. Children's learning and understanding of mathematical concepts and processes, such as number structure and relationships, number sequencing, position, or geometric properties, may be supported by experiences using their own body movements. The aim of this paper is to share classroom activities trialled within the Embodied Learning in Early Mathematics and Science project in 7 classrooms focusing on conceptual body movements. The results share what was trialled, and what was observed in relation to children's learning of mathematical processes. Findings revealed that body movement is a helpful mode through which young children can learn and communicate mathematical understanding.

Children's use of body movement supports the development of egocentric spatial frames of reference (Dackermann et al., 2017) as they explore the physical environment around them relative to their own perspective. In the context of mathematics, spatial frames of reference are important to model, and then visualise, structural aspects that are mathematically important such as the equal spacing of numbers on a number line, or the conceptual differences in positioning between 'on top', 'under' and 'next to'. *Conceptual* body movement differs from "movements for the sake of movement" (Shoval, 2011, p. 454) that are simply physical in nature, for example, children running on the spot while counting. *Conceptual* body movement involves whole-body movement that is task-specific, where actions relate directly to conceptual understandings. For example, physically jumping forward to model the process of adding-on 3 on a number line. These more purposeful actions Shoval (2011) calls 'mindful movement' and are for "the purpose of learning" (p. 454). Shoval's research highlights that learning can be enhanced when children participate in co-operative action and re-enactment using their bodies in small groups. Garrett et al. (2018) propose that "embodied representations of concepts create pedagogical opportunities to support student learning" (p. 6). The classroom provides an opportunistic space from which to observe these research claims.

During Phase 1 of the ELEMS project, the seven teachers involved in the research at the school were provided with three days of professional learning (PL) across the year. The first PL day focused on the research behind the embodied learning principles (including conceptual body movement) and examples of classroom activities the teachers could trial or adapt. This paper presents a selection of the activities the teachers' trialled or created that embedded conceptual body movement, discusses how they connect to mathematical processes and concepts, and offers ideas about student learning as identified by the teachers during these lessons.

The following questions guided the analysis of these activities: How do teachers incorporate conceptual body movement in mathematics lessons? and What potential mathematical learning connections were identified by teachers when using body movement?

Approach

The activities were implemented by teachers in Preschool, Kindergarten, Year 1 and Year 2 classrooms. The data related to the activities was either self-reported (by the teachers via the SeeSaw classroom journal, https://web.seesaw.me/, or during post-Phase 1 teacher interviews) or observed by the researchers (during weekly visits to the classrooms where some activities were co-designed by the mentoring researcher and teachers). The selected activities presented in this paper are from the Kindergarten and Year 1 classrooms (n = 4 teachers, n = 77 students). Activities linked to a range of curriculum areas such as data, position, number sense, patterning, time, and mass, see Table 1.

Table 1

Body Movement Activities Aligned to Mathematical Concepts

Number and algebra	Measurement and space	Statistics
Being the pattern—children used whole body movement to make a ‘two pattern’. *Leaping number line*—children stood in place of numbers to act out addition and subtraction number sentences. 	Being a clock—children represented the numbers around the clock face and two children were the hands 	Being the data—children used whole body to be the data points in a column graph. 
Ten frame hustle, and *Act it out*—children stood in the frame to make numbers to 10, and to depict addition and subtraction scenarios 	Miming heavier or lighter—children acted out what it might be like carrying something heavy or light. *Stand where?*—children locate themselves in a particular box to match instructions given in relation to left, right, forwards, backwards. 	
Making numbers—children used their bodies to make numbers 1 to 10 on the floor in pairs or groups. *Number track counting*, and *Before and after*—children walk along forwards or backwards 		

Titles in *italics* in Table 1 indicate activities that have been refined and are included in the PL package being developed. Some of the activities were explored by the teachers over multiple lessons within a programmed unit of work. Several of the accompanying images within this paper are from when activities were ‘recreated’ in collaboration with the researchers and teachers as part of the development of the PL package that will be utilised in Phase 2.

Findings

Incorporating Conceptual Body Movement in Mathematics

Teachers reported in post-Phase 1 interviews that they "used a lot of gesture and a lot of body movement" [Lauren] as they were "the most naturally occurring in our classroom" [Lauren]. Melissa commented that she "did do a lot of body movement … I'm using it as a trigger for things … body movement would trigger–a memory of a learning." Crystal referred to it as "full body movement" in her interview stating it was an opportunity to "just do different activities where their full bodies involved". Of the lessons teachers reported on in SeeSaw, the use of body movement as an embodied learning mode was mentioned the most (in 19 of the 40 lessons). Teachers connected body movement to a wide range of mathematical processes and concepts: "to see how the students could read, describe and interpret results" [Rosa], "to make the patterns using their bodies" [Crystal], "to find the total then move that many steps forward" [Crystal], "to explore ten frames and addition using ourselves as counters" [Melissa], "to be directed to a number and move backwards and forwards" [Rhonda], "to make the numbers 1-4 using our bodies" [Melissa], "to walk like they were heavy or light" [Melissa], "to move with their bodies towards and away from positions including forward, backward, left and right" [Isla], "to create a clock using our bodies [Crystal].

Learning Connections Identified by Teachers

In the interviews, teachers self-identified ways in which connecting mathematical processes to conceptual body movement was impacting their students' learning in positive ways. Rosa reflected that:

> You could just definitely see the improvements in them [the students] … I mean they're kids, to have them sit still for a long time, it doesn't work. So if they are up, using their body, they seem to make that connection.

Teachers provided examples of potential learning connections children made when lessons focused on conceptual body movement. Teachers reported learning gains, where "body movement enabled students to gain a deeper understanding of patterns, that they can be more than just colours repeated" [Crystal], and building children's conceptual development "to understand the concept of being straight and facing the number directly for us to be able to read the time" [Crystal]. Teachers were also able to identify potential misconceptions, "interestingly we had to correct some reversals (a huge focus on class) within body positioning" [Melissa], where assistance was needed, "students needed guidance and support to make the number line round. Spacing was mentioned by a few students" [Crystal], and a shift in confidence, "we used this line to do subtraction as well. Students who don't normally respond to questions were able to confidently answer the questions" [Crystal].

An interesting additional finding was the positive impact the project was having on teachers' pedagogical practice. Melissa reflected on how the project allowed for time to try new practices:

> I mean, we knew there's more than one way, but there's actually more than two ways and more than three ways and that that it doesn't have to be so regimented with the teaching. It gave us a little bit more freedom to experiment with new things. And you know, things like that, doing body movement for maths, is not usually something typically we might tie together, but it kind of opened that scope.

Teachers also reflected on their own understanding and interpretations of the embodied learning principles. Crystal discussed a lesson using number lines:

> Rosa and I went outside to trial a body movement lesson. Students were given a simple equation and had to find the total then move that many steps forward. Students enjoyed moving but we realised this movement was not embodied learning.

Discussion and Conclusion

Teachers were able to easily incorporate conceptual body movement as a mode of representation into daily classroom lessons. Teachers themselves reported that body movements were one of the embodied learning modes they could repeatedly enact on a regular basis. These activities were an opportunity to identify misconceptions such as number reversals by Melissa, or areas that need further explicit teaching such as number spacing related to clocks mentioned by Crystal. Incorporating body movement was an opportunity to notice conceptual understanding, to assess knowledge, and to build confidence in reluctant speakers as reported by Crystal when exploring equations on the number line. Teachers found ways to weave the embodied learning principles (specifically body movement) into curriculum lessons utilising environmental spaces inside and outside the classroom as well as making use of physical mathematical structures such as number tracks, number lines, ten frames, and grid-structured classroom mats to assist students in developing spatial frames of reference.

Two teachers questioned whether or not the activities they were implementing aligned with conceptual body movement. Crystal's reflection in the number line lesson she and Rosa completed together is evidence of this self-reflection. The students' movement in the positive direction *is* related to the mathematical process of addition, therefore the activity *does* relate to conceptual boy movement. Nonetheless, observing teachers wrestle with the concept of conceptual body movement indicates their attention to making the connections between mathematical process and conceptual body movement correct, and explicit.

Input from the teachers was invaluable in creating and refining the activities to ensure they aligned to age-appropriate classroom practice as well as the theoretical framing of conceptual body movement. Similar to Garrett et al.'s (2018) findings, implementing activities that focused on conceptual body movement, "impacted teachers' pedagogical practices in various ways" (p. 9), where immediate changes were voiced by the teachers themselves. These initial findings may show "significant promise for improving students' learning engagement in mathematics as well as professional renewal for teachers" (p. 16) through the use of embodied learning principles. A future research direction might include observing when/if students use impromptu body movements as a thinking tool about the mathematical concepts, or do they choose to initiate body movements, without prompt by the teacher, as a communication tool.

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Finger Tracing, Noticing Structures and Drawing

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This paper presents an initial analysis of 10 Preschool children's responses to a look-draw-trace-draw task. The findings suggest that figure-tracing helped half the children to produce a more accurate representation of the geometric figure presented to them, in their second drawing.

The development of children's mathematical drawing capabilities is largely determined by developmental factors that span several years. Natural development of drawing abilities from playful scribble to realistic representations of imaginings and external objects takes time (Machón, 2013), and is linked with both cognitive and motor factors. Hand-motor control is a crucial component of drawing skill and develops over time in young children (Cohen, Bravi, & Minciacchi, 2021). Cognitive flexibility and associated drawing flexibility (ability to adapt and change familiar figurative schemas) increases over time and with age (Ebersbach, & Hagedorn, 2011). To be able to enhance mathematical drawing of children, particularly those developing at slower rates than expected, teachers need strategies that produce positive outcomes in a shorter timeframe.

In mathematics education, children might be asked to use drawing as a representation of their thinking (an external representation of an internal representation), or to produce a record of tangible objects (external representation of an external representation). Representing a visible, external model through drawing is a different task to drawing an object from an internal image or graphic schema. To reproduce the appearance of an object, say a geometric figure (e.g., a 2D shape), the child needs to give attention to, or notice, the key characteristics of the figure. Therefore, strategies that help the child focus their attention and raise their awareness of task demands are likely enhance the child's drawing performance (Morra, 2005; Sutton & Rose, 1998). This line of thinking suggests that increasing children's 'noticing' might have an immediate effect on children's drawing reproduction accuracy, if other developmental factors are sufficiently advanced. Pointing and finger tracing techniques have been shown to increase performance in particular mathematical tasks in older children (E.g., Hu, Ginns & Bobis, 2015) and may assist children to attend to spatial or structural features of a figure. While pencil-tracing might also be helpful, finger-tracing evokes the genetically driven visual-attention response to pointing (Hu, Ginns & Bobis, 2015), and contact with the surface activates the sense of touch and hence a different part of the brain to 'looking' only.

For this paper we pose the question, what changes in the preschool children's drawings occur after finger-tracing a figure?

Method

Context and Participants

This study was imbedded within the Embodied Learning in Early Mathematics and Science (ELEMS) project which involved Preschool to Year 2 teachers and their classes in one school. Although data for this tracing-drawing study was collected from all four cohorts, only the preschool data has been tentatively analysed at this point. The 10 Preschool children (approx. 4 ½ years) with parental permission to participate are the focus of this paper.

Procedure

a) In an individual task-based interview, the child was shown a geometric figure (see Figure 1) and invited to look carefully then draw what they saw. As soon as the child began drawing, the card was turned facedown so the figure was hidden.

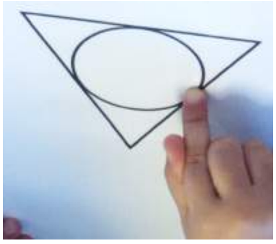


Figure 1. Child finger-tracing the geometric figure.

b) When the drawing was completed, the figure was again placed in front of the child and the interviewer asked them to trace around the shapes with their finger. To ensure the child understood the instruction, the interviewer demonstrated by pointing to the top left corner of the triangle and touch-tracing across the top of the triangle, around the corner (clockwise direction), then invited the child to trace it themselves. If the child did not automatically also trace around the circle, they were prompted to do so, again beginning the trace at the top and moving clockwise.

c) The child was then invited to draw the figure again, on a new sheet of paper, and the figure was again hidden from view.

Analysis was exploratory and open-ended and used several approaches to examine both the product (finished drawings) and process (video of drawing actions). The pairs of drawings for each child were compared for changes and annotated with arrows and numbers to indicate the drawing process. Observation notes were added to capture some key changes, features, or additional information from the videos. The pairs of drawings were grouped according to the magnitude of change between Drawings 1 and 2.

Findings

The drawings were idiosyncratic, both in process and product, with few patterns identifiable in the small sample. Some observations are:

- Half the pairs of drawings show a definite change in structure and detail in the second drawing (See Figure 2). A notable change for P107 is from drawing two separate shapes to one shape enclosed inside another. Another significant change is from a single stroke to a closed shape (P116).
- Three pairs of drawings showed minimal changes, but the first drawings were already well formed. Small changes were a slightly larger circle or slightly 'pointier' triangle corners. (See Figure 3).
- Two children produced highly idiosyncratic pairs of drawings that were very different the drawings of the other children (See Figure 4).
- All children drew a closed shape (P116 only after tracing), and most were recognisable as a triangle. Most drew a recognisable circle inside.
- No child succeeded in drawing a circle that touched all three sides of the triangle, though P103 tried to make such an adjustment in her second drawing.
- All children except P112 drew the outer shape (triangle) first using a continuous line.
- The starting point and direction of drawing varied. Although some children changed this in the second drawing, it did not seem to be influenced by the tracing sequence modelled by the interviewer.
- Two children persisted with drawing the triangle upside-down relative to the figure presented to them (P213, P109).

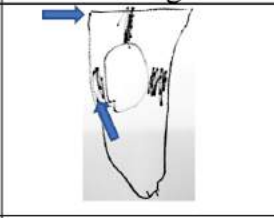
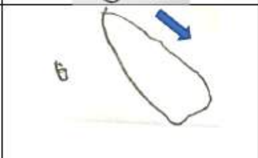
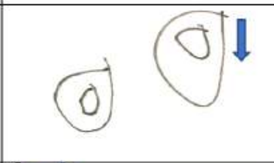
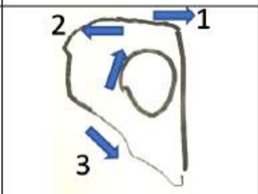
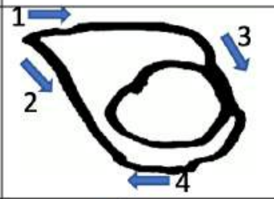
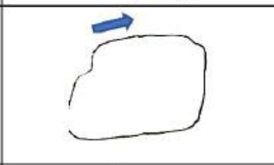
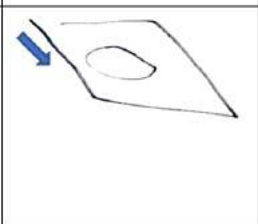
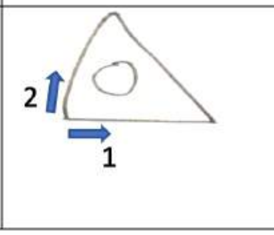
Student code	Look then draw. Drawing 1	After finger trace Drawing 2	Notes
P103			Explained scribble by saying, "I wanted it to touch".
P107			From two separate shapes to one shape enclosed by another. Continued to fill the page with the same figure,
P110			Awkward pen grip. Very slowly drew separate pieces. Orientation unclear. Inner circle enlarged.
P116			Poor pen grip. Needed coaching to trace all around. From a single stroke to a closed shape
P213			Immediately said, "It's like a triangle with a circle", but drew 4-sided shape. Changed from 4 sides to 3 sides. Incorrect orientation of triangle.

Figure 2: Before and after drawings that show change.

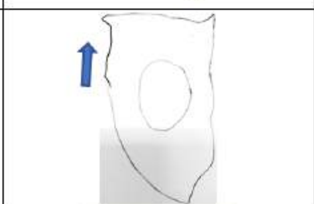
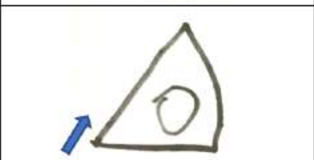
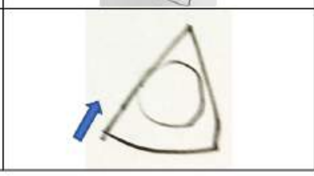
Student code	Look then draw. Drawing 1	After finger trace Drawing 2	Notes
P104			Minimal change
P108			Started in middle of left side. Drew quickly. Lack of straight lines. Change to sharper corners.
P109			Maintained incorrect orientation of triangle. Circle larger.

Figure 3. Before and after drawings with minimal change.

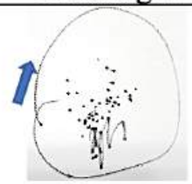
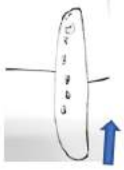
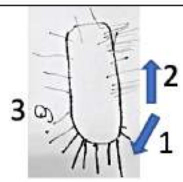
Student code	Look then draw. Drawing 1	After finger trace Drawing 2	Notes
P112			Drew circle first, then enclosed with 'triangle'. Needed prompting to trace circle, but did so anticlockwise. Quickly drew a large circle then giggled
P113			Could not follow sides of triangle all the way around when tracing. Drew large shape, then small 'curl' outside, then added strokes.

Figure 4. Highly idiosyncratic drawings.

Discussion and Conclusion

With the limitations of this small exploratory study in mind, we offer some speculative interpretations of the findings. One explanation for the minimal-change in drawings (Figure 4) is that these children held stable mental images of triangles and circles and the task evoked existing schema (Ebersbach, & Hagedorn, 2011) useful for drawing the composite figure. P110 and P116 (Figure 3) struggled with a lack of fluidity in hand movement but the role this played in how they responded to the tracing experience is unclear (Cohen, Bravi, & Minciacchi, 2021). Both P107 and P116 exhibited a remarkable change in geometric structure, in a topological sense, by moving from separate shapes to enclosed shapes, and a single line to a closed shape respectively. It seems likely that the act of tracing stimulated this change somehow. The odd second drawing produced by P112 can be accounted for as playfulness. P113 produced drawings that are classic examples of the drawing stage of exploring combinations of forms, typical around age 3 years (Machón, 2013), which suggests the child was not ready for the type of drawing task used in this study.

In conclusion, the 'self-correction' of drawings by half the children suggests that the finger-tracing may have supported these children's noticing of the structure of the figure presented to them. The preliminary findings from this small sample provide encouragement for continuing the exploratory analysis with the data from the 5- to 8-year-olds and refining the analysis techniques in preparation for further studies, in which the role of memory in 'hidden figure' tasks should be considered.

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Changes in Year 2 Children's Drawings of a Subtraction Story

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There is an educational expectation that children's natural drawing will develop into proficient mathematical representations and formal diagrams, yet there is little research available to guide the assessment and development of children's mathematical drawing skills. The aim of this paper is to explore how Year 2 children (approx. 7 years) chose to represent their interpretations of a simple story that is suggestive of the take-away subtraction process, and what changes occurred when the drawing task was repeated 6 months later. Analysis of 13 pairs of drawings revealed changes in what the children drew (categories of number representations) and how they drew it (style). The findings suggest that substantial change in children's representational ability can occur in within 6 months.

Children's representational competence in drawing has been linked with cognitive maturity and flexibility (Brooks, 2009), particularly regarding mathematical development. Children's drawing is also a source of evidence for internal "processes of notational competence and representational change" (Karmiloff-Smith, 1990, p. 58). Although drawing is a naturally developing ability in young children (Brooks, 1990) it can also be influenced by environmental factors including adult interactions (Malanchini et. al., 2016), making drawing development pertinent to teaching practice. Indeed, supporting children's development of drawing schemas, particularly dynamic schematisation (depicting movement and change) can enhance both drawing skill and mathematics comprehension (Poland & van Oers, 2007).

In the context of the Embodied Learning in Early Mathematics and Science project, the Pre-school to Year 2 teachers at one school explored supporting the development of children's drawing through increasing the opportunities for children to draw, discuss their drawings and experience some teacher-modelling of ways of drawing mathematical objects and processes. Pre-school to Year 2 students completed the 'Birds drawing task' in May 2022 (Time 1) as part of a larger assessment of drawing development requested by the teachers. In December 2022 (Time 2), an opportunity arose to repeat the drawing task with participating students. The 'Birds drawing task' is a very brief story used as a provocation to draw (Way, 2018), which is suggestive of a subtraction process.

This paper is focused by the questions: How do Year 2 children represent through drawing, the subtraction process implied by a simple 'take-away' story? and What changes in drawings are evident after 6 months?

Procedure

The task instructions for the 'Birds drawing task' were delivered verbally to the group of children.

> Say: 'Listen to this little story. Then I'm going to ask you to draw what happened.'
> 'Five birds sat in a row along the top of a fence. Two birds flew away.'
> Repeat the story, then ask them to, 'Draw what happened in the story'.

Only the data from Year 2 students is used in this paper, as an initial development of the analysis technique. In one of the Year 2 classes, 13 students were present for both Time 1 and Time 2 of the drawing task, and these 26 drawings are the subject of this paper.

The modelling of the 'take-away' subtraction process can be described as a sequence of three steps: 1. Represent the original quantity in a group, 2. Separate or 'take-away' the relevant number items, 3. Determine the number of items remaining. Steps 1 and 2 are dynamic—requiring movement of some type. Step 3 implies some form of acknowledgement of the result of the process.

Two approaches were used in the analysis of the drawings. The first approach involved sorting the Time 1 drawings into categories according to whether they depicted one, two or three steps in the subtraction process, or no steps. The process was repeated for the Time 2 drawings. The second approach involved comparing the two drawings produced by each student and examining the nature of changes in the style of the drawing.

Findings

The Drawing Categories

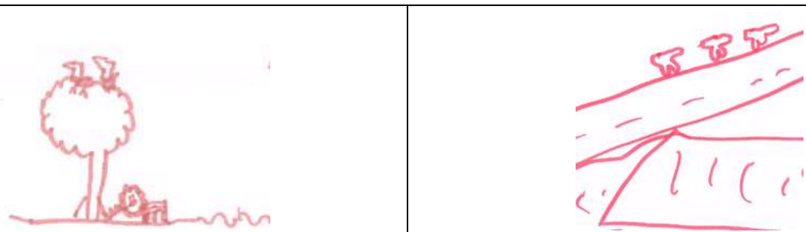
The categories are presented in order of the completeness of the depiction of the subtraction process, ranging from non-depiction of any step in the subtraction process, to depiction of all three steps.

Category 1 non-depiction. The drawing does not depict any recognisable numerical information from the 'story', nor suggest any part of the subtraction process (Figure 1).

Multiple items that do not match any of the group quantities. (2C01-Time 1)

Figure 1. Example of non-depiction.

Category 2 one step. One step of the 3-step process of take-away subtraction is drawn: the 5 original birds, or the 2 that flew away, or the 3 that remained, with sub-categories identified based on the specific number of birds depicted (Figure 2)



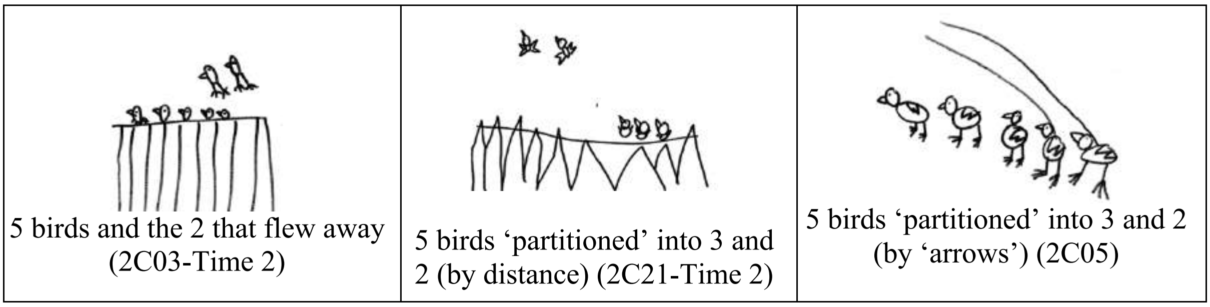
Group of 2 (2C13–Time 1)

Group of 3 (2C20-Time 1)

Figure 2. Examples for each sub-category of one step drawings.

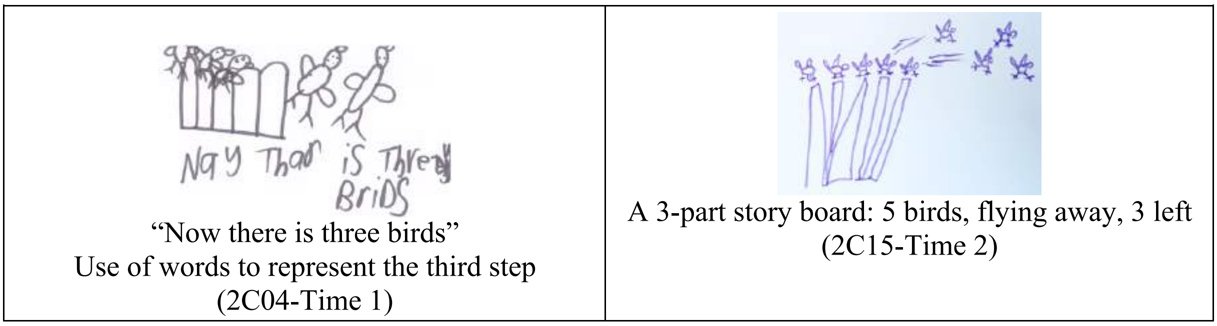
Category 3 two steps. Two of three steps are drawn depiction either 5 birds and the 2 that flew away (total of 7 birds), or the group 5 birds is partitioned into groups of 3 and 2. The partitioning is typically represented by separation of the subgroups by distance but may involve crossing out of 2 birds or arrows/lines indicting movement away (Figure 3).

Category 4 three steps. Some drawings included a strategy for focusing on the remaining 3 birds, as well as depicting the original group of 5, and the 'taking away' of 2 birds, even though the 'story' did not mention the remaining group of three, nor ask for 'how many left?'. Three-step drawings complete the operation of take-away subtraction and could be construed as also representing the equation 5 - 2 = 3 (Figure 4).



5 birds and the 2 that flew away (2C03-Time 2) | 5 birds 'partitioned' into 3 and 2 (by distance) (2C21-Time 2) | 5 birds 'partitioned' into 3 and 2 (by 'arrows') (2C05)

Figure 3. Examples of two steps drawings.



"Now there is three birds"
Use of words to represent the third step
(2C04-Time 1)

A 3-part story board: 5 birds, flying away, 3 left (2C15-Time 2)

Figure 4. Examples of strategies for depicting the three steps.

Changes in Category and Style

No child drew the same drawing both times with changes in style and/or changes in the parts of the subtraction process they chose to depict (category change). Table 1 shows the distribution of students' drawings across the categories. Examining the table to match the student codes in the Time 1 and Time 2 columns reveals changes in categories by individual students. Most drawings from both Time 1 and Time 2 depicted the two steps in the story, and hence two steps in the subtraction process.

Table 1

Distribution of Students' Drawings Across the Categories, for Time 1 and Time 2

Approach Category	Sub-Category	Time 1 Drawings—May (Student codes) N=13	Time 2 Drawings—December (Student codes) N=13
Non-depiction		2C01*	
1 step	5 birds	2C14*	2C14
	2 birds	2C13	
	3 birds	2C10 2C20	
2 steps	5 and 2 (total 7)	2C05 2C06* 2C15	2C03 2C06* 2C04
	3 and 2 (partition)	2C08* 2C17 2C21*	2C01 2C08 2C13 2C20 2C05 2C10 2C17 2C21
3 steps	5 - 2 = 3	2C03 2C04	2C15

*Signifies an incorrect number of items drawn (e.g., 6 birds rather than 5).

A noticeable shift in distribution in Time 2 is towards 2-step drawings that show the partitioning of the group of 5 into subgroups of 3 and 2, rather than representing two quantities specified in the story (5 and 2), or only one of the groups from the story.

Only three students did not substantially change their style of drawing. About half the students produced changes in both category and style (For example, Figures 5 & 6).

Most changes in style involved a more mature representation of the birds, showing some distinctive characteristics such as body shape, as can be seen in Figure 5. The most striking change in style was produced by student 2C01 (Figure 6) with a change from a drawing lacking any features of the story, to a drawing that shows five birds (circles) with two crossed out.

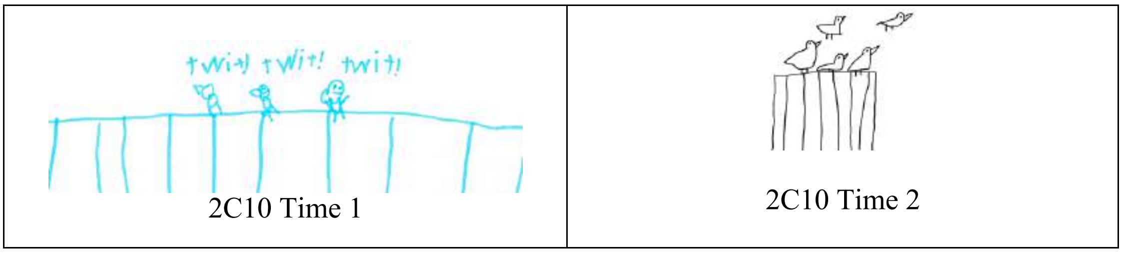


Figure 5. Example of change in both style and category.

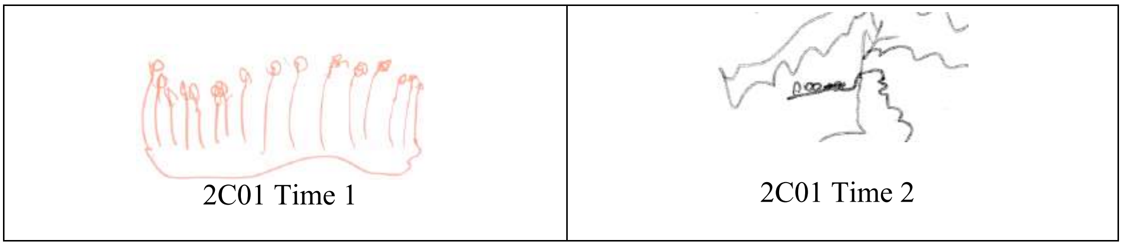


Figure 6. Example of change in both style and category.

Discussion and Conclusion

It is important to note that the story-task was not intended as an assessment of the children's knowledge of subtraction, but rather and an opportunity to study how they responded. Using the categories related to the steps in the take-way process revealed that, in 6 months, the children had an increased tendency to mathematise the story and represent the partitioning of a group of five into groups of two and three. Comparing the pairs of drawings showed a shift in representational maturity. These findings contrast with the relative stability in 'human figure' drawing over 6 months found by Malanchini et.al (2016). Although no direct claim can be made about the role played by the ELEMS project teachers' increased attention to drawing development, the results do illustrate that substantial development in mathematical drawing skill can occur within 6 months. The analysis procedure will now be applied to the full collection of Preschool to Year 2 drawings to explore age-related patterns and other relationships between Time 1 and Time 2. Further research is needed to develop drawing tasks and interpretation guidelines that teachers can use to monitor their students' drawing development and support development of mathematical drawing ability.

References

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